CRACK PROPAGATION IN UNORIENTED AMORPHOUS POLYMERS

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The initial direction of crack development is investigated experimentally as a function of the ratio of the stress intensity factors $\lambda = k_2/k_1$ at its vertex for large λ. The investigations were performed by the photoelasticity method on a photoelasticity meter of the firm "MEORTA" (Czechoslovak Socialist Republic) on 4-mm-thick specimens of ÉD-6 epoxy resin with maleic anhydride as solidifier (Fig. 1). Cracks were produced in the specimens by using a special knife for which the specimens were heated to 83°C at which they softened and were easily pierced by the knife. Reduction of the internal stresses was accomplished by annealing at 90°C for 30 h with subsequent slow cooling at a rate of 5°C/h. The stress field was produced at the crack vertex by loading the specimen in a stretching apparatus. In the general case of loading in the stress state, the stress field at the crack vertex is described by the following equations [1]:

$$\begin{aligned}\sigma_x &= \frac{k_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\cdot\sin\frac{3}{2}\theta\right)-\frac{k_2}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cdot\cos\frac{3}{2}\theta\right);\\ \sigma_y &= \frac{k_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\cdot\sin\frac{3}{2}\theta\right)+\frac{k_2}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\cdot\cos\frac{3}{2}\theta;\\ \tau_{xy} &= \frac{k_1}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\cdot\cos\frac{3}{2}\theta+\frac{k_2}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\cdot\sin\frac{3}{2}\theta\right).\end{aligned}\tag{1}$$

Different ratios between the intensity factors were obtained by varying the slope of the crack α and its length H (their values are presented in Table 1). The ratio between the intensity factors is determined from the equation [2]

$$\lambda^2-(4/3)\lambda\operatorname{ctg}2\theta_m-1/3=0,$$

where θ_m is the angle between the crack direction and a ray connecting the crack vertex to the most remote point on the isochrome loop. The value of θ_m was measured by means of visually distinguishable isochromes nearest to the crack vertex, and hence the error in measurement did not exceed 8%.

It has been shown in [3] that crack development in polymethylmethacrylate occurs at large angles in the direction of the gradient of the maximum tangential stresses at its vertex. Investigations performed showed that λ is independent of the magnitude of the applied loads; hence, it is sufficient to determine the isochrome pattern for any load not exactly equal to the fracture load in order to obtain information about the dependence of the crack propagation direction on the ratio between the intensity factors.

The dependence of the initial crack propagation direction on the ratio between the intensity factors is shown in Fig. 2 (curve 1). It is shown that an abrupt change in the angle of rotation of crack propagation occurs for $\lambda > 1$.

It is interesting to compare the results obtained with results of a theoretical approach to the determination of the initial direction of crack growth. Two approaches exist at this time [4].

1. The direction of crack growth is determined by the orientations of areas on which the maximum tensile stresses are achieved. The expression for the angle of orientation of the area of maximum tensile stresses is obtained from the condition $\sigma_\theta' = 0$. The solution of this equation for the angle θ_c has the form

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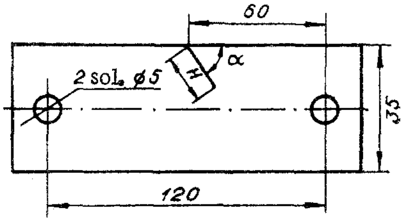

Fig. 1

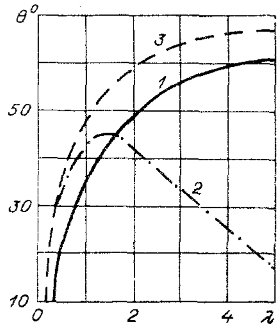

Fig. 2

TABLE 1

α°	15	30	45	60
M, mm	5; 10	5; 15	5; 10	2,5; 5; 10

$$\theta_c = 2\,\mathrm{arctg}\,\frac{1-\sqrt{1+8\lambda^2}}{4\lambda}.$$

2. It is assumed that the angle θ_e at which the initial crack growth occurs is a root of the equation

$$\Gamma(\theta) = 2\gamma(\theta),$$

where $\Gamma(\theta)$ is the vector of the energy flux through the crack vertex, and $\gamma(\theta)$ is the magnitude of the specific energy expenditure in crack propagation. If it is assumed that γ is independent of θ and the loading history, the expression for the initial crack growth direction becomes

$$\theta_e = -\,\mathrm{arctg}\,[2\lambda/(1+\lambda^2)].$$

According to the approaches mentioned, angles of crack growth were computed for values of λ obtained experimentally (see Fig. 2, curves 2 and 3). Comparing them to experiment shows that the force approach corresponds well to experimental data for $\lambda > 1$. The most probable explanation of this fact can be given on the basis of the kinetic theory of the strength of solids [5]. The stress field described approximately by (1) originates upon application of a load to a specimen with a crack near its vertex. As the load grows, a loss in the capacity of the material to plastic deformation and the formation of a narrow domain whose edges are fastened by noninteracting molecular filaments occur in front of the crack growth on areas with maximum tensile stresses. As the load increases further, the molecular filaments near the crack vertex break. The crack growth therefore occurs according to the force approach in the direction of orientation of areas with maximum tensile stresses.

The unacceptability of the energy approach in the form elucidated above is explained by the formation of a region with oriented structure near the crack vertex in an unoriented amorphous polymer. Hence, the assumption of independence of the magnitude of the specific energy expenditure from the direction of crack growth is invalid.

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STABILITY OF COMPRESSED VISCOELASTIC ORTHOTROPIC SHELLS

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The stability criterion for structures operating under creep conditions, which is based on comparing the unperturbed and perturbed motion trajectories, is proposed in [1, 2]. On the basis of this criterion, the singularities in the behavior of compressed viscoelastic thin-walled shells are analyzed in this paper.

The equilibrium and strain compatibility equations of thin-walled shallow shells with the interlayer shear taken into account according to the Timoshenko hypothesis are written in the form [3, 4]

$$
\begin{gathered}
h\left[\mathbf{\Gamma}_{3131}(\gamma_{1,1}+u_{3,11})+\mathbf{\Gamma}_{3232}(\gamma_{2,2}+u_{3,22})\right]+\left[\frac{1}{R_{ij}}+\left(u_3+u_3^0\right)_{,ij}\right]N_{ij}=0,\\
\mathbf{D}_{1111}\gamma_{1,11}+\mathbf{D}_{1122}\gamma_{2,12}+\frac{1}{2}\mathbf{D}_{1212}(\gamma_{1,22}+\gamma_{2,12})=h\mathbf{\Gamma}_{3131}(\gamma_1+u_{3,1}),\\
\mathbf{D}_{2211}\gamma_{1,12}+\mathbf{D}_{2222}\gamma_{2,22}+\frac{1}{2}\mathbf{D}_{1212}(\gamma_{1,12}+\gamma_{2,11})=h\mathbf{\Gamma}_{3232}(\gamma_2+u_{3,2}),\\
\frac{1}{h}\left[\mathbf{K}_{1111}F_{,2222}+2\left(\mathbf{K}_{1212}+\mathbf{K}_{1122}\right)F_{,1122}+\mathbf{K}_{2222}F_{,1111}\right]=-e_{ik}e_{jl}\left\{\frac{1}{R_{kl}}u_{3,ij}+\right.\\
\left.+\frac{1}{2}\left[\left(u_3+u_3^0\right)_{,kl}\left(u_3+u_3^0\right)_{,ij}-u^0_{3,kl}u^0_{3,ij}\right]\right\}\qquad(i,j,k,l=1,2),
\end{gathered}
\tag{1}
$$

where γ_i are the angles of rotation of the normal to the middle surface; u_3, u_3^0 are the additional and initial shell deflections; F is a function of forces acting in the middle surface, $N_{ij}=e_{ik}e_{jl}F_{,kl}$; h, R_{ij} are the thickness and radii of curvature ($R_{12}=R_{21}=\infty$); $\mathbf{K}_{ijkl}$, $\mathbf{\Gamma}_{ijkl}$ are operators of the form

$$
\begin{gathered}
\mathbf{K}_{ijkl}f=\frac{1}{E_{ijkl}}f(t)+\int_0^t K_{ijkl}(t-\tau)f(\tau)\,d\tau,\\
\mathbf{\Gamma}_{ijkl}f=c_{ijkl}f(t)-\int_0^t \Gamma_{ijkl}(t-\tau)f(\tau)\,d\tau;
\end{gathered}
$$

E_{ijkl}, c_{ijkl} are elastic constants; $K_{ijkl}(t-\tau)$, $\Gamma_{ijkl}(t-\tau)$ are the creep and relaxation kernels which are invariant relative to the origin:

$$
\begin{gathered}
0\leqslant\int_0^\infty K_{ijkl}(\tau)\,d\tau=K_{ijkl}<\infty,\quad 0\leqslant\int_0^\infty \Gamma_{ijkl}(\tau)\,d\tau=\Gamma_{ijkl}<1,\\
\mathbf{D}_{ijkl}=\frac{h^3}{12}\mathbf{\Gamma}_{ijkl},\quad e_{ik}=\begin{cases}1, & i>k\\ 0, & i=k\\ -1, & i<k.\end{cases}
\end{gathered}
$$

Here and henceforth the summation is over the repeated subscripts. The subscripts following the comma denote differentiation with respect to the appropriate coordinate. The x_1, x_2 axes coincide with the lines of principal curvature and the axes of viscoelastic symmetry, while the x_3 axis is perpendicular to them and directed toward the center of curvature.

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